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MODIFIED TOPCAP OBJECTIVE FORCE STRUCTURE MODEL

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number) Spiralling personnel costs, coupled with increased emphasis on management by objectives, has accentuated evaluation of personnel quality/quantity/cost tradeoffs pertinent to the development of a career force objective and enlisted force management system. A modification to one of the methodologies in the Total Objective Plan for Career Airman Personnel (TOPCAP) Computerized Management System, the Objective Force Structure Model, is proposed. The effort was prompted by a need for a method to evaluate tradeoffs between the "quality" of personnel entering the career force and the effects on the TOPCAP objective grade and force configuration.		

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PREFACE

This research was initiated under Project 6323, Personnel Management Research and Development; Task 632302, Research and Development on Mathematical/Econometric Models of Air Force Personnel System; Work Unit 63230215, TOPCAP Simulation, which has been terminated. The research has been completed under Project 2077, Personnel and Manpower Management Systems Development; Task 207703, Computer-Based Models of Air Force Personnel Subsystems; Work Unit 20770307, TOPCAP Simulation

The author is indebted to Captain J. Sears (AF/DPMDW) for the example stated in footnote 6. AF/DPMDW plays a major role in exercising and updating TOPCAP computer-based methodologies.

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MODIFIED TOPCAP OBJECTIVE FORCE STRUCTURE MODEL

I. INTRODUCTION

Background¹

The U. S. Air Force Personnel Plan identifies management and structural objectives for each component of the personnel force. A portion of that plan delineates airman management objectives in life cycle terms: procurement, education and training, utilization, sustainment, and separation and retirement. These objectives provide a management system directed at achieving and maintaining an optimum airman force and grade configuration. The Total Objective Plan for Career Airman Personnel (TOPCAP), as described in the U. S. Air Force Personnel Plan, Volume III, identifies the optimum airman force and grade structure.

TOPCAP is an Air Force response to a 23 December 1968 memorandum from the Assistant Secretary of Defense, Manpower and Reserve Affairs (OASD/M&RA), which provided the military services with guidance in the development of enlisted force management systems. The principle objectives were to:

- a. Identify within each service groupings of specialties which will be useful for both management and reporting functions, and which will serve as a communications vehicle between elements of OSD and the individual services.
- b. For each grouping establish compatible and attainable manpower and personnel management goals that reflect full consideration of current and long range operational requirements.²

CASD guidance required the specialty groupings to be self-renewing, with a visible and logical promotion pattern; to possess qualifications, in terms of potential abilities and aptitudes, so as to allow training and assignment of personnel from one specialty to another within a grouping; and to allow limited lateral movement among groupings, requiring a first term base both in terms

of quality and quantity which will sustain first term requirements as well as replace career force losses. For each of the specialty groupings, the Air Force was required to establish a number of management goals.

It was necessary to define an objective, for each specialty grouping, which reflected a career force meeting current and long range requirements, based on achievable retention patterns and obtainable at a reasonable total systems cost. An additional requirement was to establish a desired grade distribution by year of service. TOPCAP is the airman grade and force structure objective, and the Air Force enlisted force management system is a complex interaction of management objectives, goals, and concepts which assist personnel managers in defining, attaining, and maintaining the TOPCAP objective.

The Problem

In addition to accepting TOPCAP as a response to enlisted force management guidance, OASD(M&RA), in a 17 May 1971 memorandum to the Assistant Secretary of the Air Force (M&RA), urged the Air Force to "... investigate alternatives that will produce even greater dollar savings than TOPCAP and identify tradeoffs involved in their achievement."³ Spiralling personnel costs and transition to all volunteer armed forces, coupled with recruitment and remuneration flexibilities, warranted analyses of personnel quality/quantity/cost tradeoffs pertinent to the development of a career force objective and enlisted force management system.

The development of the TOPCAP configuration, as well as guidance in force planning and programming necessary to attain and maintain the TOPCAP objective, is facilitated by the TOPCAP Computerized Management System. These modular computer-based methodologies may be exercised independently or sequentially and provide, within an occupational framework, grade and force structure definition in terms of quantity of personnel. Volume VI and Annexes to Volume III, of the U. S. Air Force Personnel Plan, discuss this activity in detail.

This report is the first of two technical reports which analyze the role of "personnel quality" in the establishment of enlisted force structure objectives. It advances a method for introducing

¹ Background information throughout is taken principally from the USAF Personnel Plan.

² Volume Three, Airman Structure Annexes (Draft), p. A-12.

³ Volume Three, Airman Structure Annexes (Draft), p. A-17.

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"quality considerations," beyond the occupational context, into the TOPCAP Computerized Management System. Specific modifications to the TOPCAP Objective Force Structure Model are presented.

II. METHOD

Choice of Model

The principal research objective was to ascertain the feasibility of introducing quality considerations to TOPCAP computer-based methodologies, while insuring ease of implementation by maintaining the basic assumptions, concepts, and philosophy involved in development of the TOPCAP configuration. In TOPCAP, the airman force is comprised of two components, the career force and the first term force. The total airman force size in TOPCAP is represented by a force range allowing for fluctuations in end-of-year strength in a peacetime environment, as well as an expansion capability for limited war contingencies. The basic concept is that a career force configuration can be determined which will remain stable or constant even though fluctuations in total force size occur within the established force range. The number of superintendents and supervisor/technicians needed in the career force to form a training cadre to provide immediate expansion to the upper level of the force range is induced. Within an occupational context, the minimum number of journeyman careerists essential to sustain the 7 and 9 skill level requirement is determined.

In order to preserve these basic premises, including quality as a modeling dimension made it necessary to extend the occupational analysis, while adhering to force size and superintendent and supervisor/technician limitations. TOPCAP philosophy recognizes training and experience as the primary determinants of superintendent and supervisor/technician skill levels. In addition, the "Requirements for airman supervisors/technicians and superintendents (7 and 9 skill levels) vary by occupation. The time required for an airman to qualify for advanced levels of skill varies not only with the individual, but with occupation as well

Further, airmen leave the service at different rates, depending upon their occupation, amount of service, and pay grade."⁴ Within these restrictions, the effects of quality (individual abilities) could be reflected in two measures. First, the quality of individuals recruited and subsequently entering the career force could affect the minimum number of journeyman careerists essential to sustain the established 7 and 9 skill level requirement; and, secondly, the distribution of the career force by year of service could be affected.

The TOPCAP Objective Force Structure Model was selected as the means of introducing quality considerations within the limits already described. The role of the Objective Force Structure Model, both in development of the TOPCAP configuration and in current airman programming actions, is reflected in Figure 1. Based on an established force range, manpower authorizations, budgetary constraints, and skill projection techniques, the number of 7 and 9 skill level requirements is determined by occupational grouping and inputted, along with "objective" attrition and upgrade rates, to the Objective Force Structure Model. The model's output takes the form of a career force objective (number of personnel and distribution by year of service) and is inputted to other methodologies in the TOPCAP Computerized Management System. Utilization of the model proceeds in two directions. First, it is used in conjunction with other models to develop the TOPCAP grade and force structure objective, as well as TOPCAP policies such as promotion opportunities. The resulting "optimum objective" represents an ideal state, however, and output from the Objective Force Structure Model is additionally used as a constraint in current airman programming actions essential to operate, subject to present restrictions, while transitioning today's force to the TOPCAP objective force configuration.

Objective Force Structure Model⁵

The Objective Force Structure Model characterizes a static career force objective, by year of total active federal military service (TAFMS), for each career field subdivision (3 digit specialty code) in the airman classification structure. The concept of the model is displayed in diagrammatic fashion as the series of illustrations within the dotted portion of Figure 1.

For each career field subdivision (CFS), the relative distribution of 7 (supervisor/technician) and 9 (superintendent) skill level personnel is

⁴ Volume Three, Airman Structure (TOPCAP), p. 1.3.

⁵ Taken, in part, from "Annex D Models and Methodologies," Volume Three, Airman Structure Annexes (Draft).

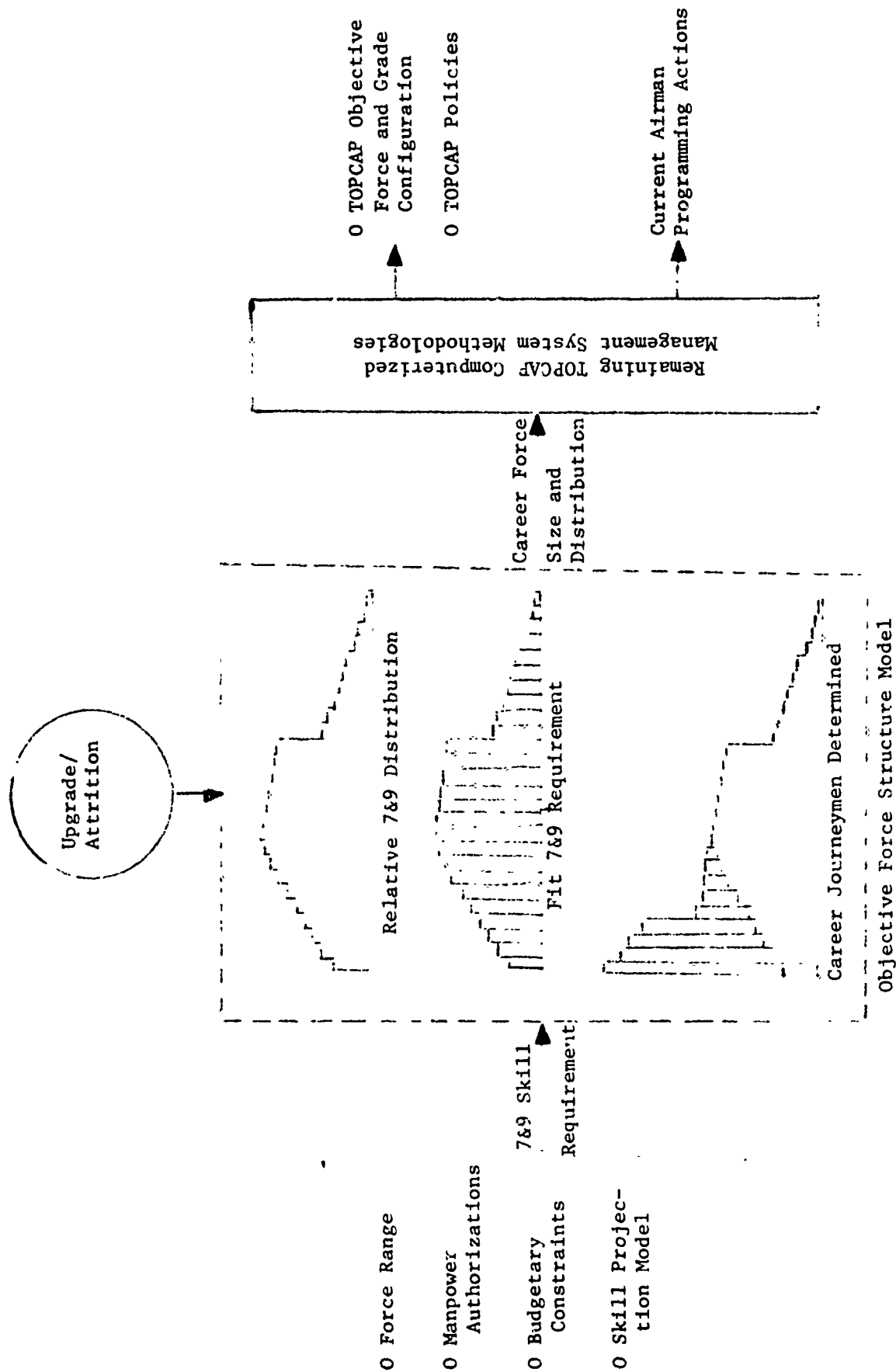


Figure 1. Objective Force Structure Model.

defined. Operationally, the distributions are based on empirical data reflecting the time required for airmen to reach the supervisor/technician skill level, as well as data representing attrition from the Air Force, by year of TAFMS, for each CFS. The number of 7 and 9 skill level personnel desired is specified. That quantity is adapted to the defined distribution resulting in a frequency distribution of 7 and 9 level personnel, by year of TAFMS. The number and distribution of career journeymen needed to sustain the 7 and 9 distribution is postulated, based on historical attrition data.

The concept of the model includes a number of implicit assumptions. This aspect of the model is discussed in Section III. Appendix A includes a mathematical description of the concept discussed in this section.

Modification

In essence, the modification to the Objective Force Structure Model is one of disaggregation. It permits analysis beyond the occupational grouping (CFS), while still effecting a solution in terms of an objective force structure profile based on a given 7 and 9 skill level requirement for the CFS. The change rests on the premise that there exist identifiable subgroups within the CFS which, because of significantly different upgrade and attrition patterns, would result, depending on the subgroup mix, in alternative objective force profiles. The concept of the modified model is shown in Figure 2.

Subgroups are identified (e.g., college graduates, high school graduates, non high school graduates) which are mutually exclusive and exhaust the CFS. For each of the subgroups, the relative distribution of 7 and 9 level personnel (by year of TAFMS) is defined. These distributions could be based on empirical attrition and upgrade data for the different groups. The defined relative distributions, combined with the CFS and 7 and 9 requirement and the specification of the subgroup mix inputted to the CFS in year 1 or 5, are converted to formulate a system of simultaneous linear equations. The solution to the system of equations results in calculation of the career force objective: the distribution of the 7 and 9 skill level CFS requirement, by year of TAFMS, and determination of the number and distribution of career journeymen needed to sustain the 7 and 9 requirement. Appendix A provides a mathematical description of the process of formulating the system of equations and calculation of the objective force structure.

III. DISCUSSION

Model Assumptions

The basic and modified Objective Force Structure Model postulate a closed and stationary objective career force by year of TAFMS and CFS. The flow of personnel is such that, while attrition can occur at each year of service, entry into the career force can take place only at the fifth year point. No allowance for cross training or entry of prior service personnel, subsequent to the fifth year, exists. By assumption, the variables are independent of the passage of time and result in a static model describing a stationary state. A stationary state is described when the model's variables take on a set of values such that the model will repeat itself unchanged indefinitely.

Personnel requirements are inflexible in the basic model. There is no deviation from the specified supervisor/technician and superintendent requirement, and empirical data resolves the number and distribution of journeyman careerists. Since the total force range is predetermined, the remainder of the 5 skill level personnel is reflected in the first term composition. The modified model retains the fixed 7 and 9 requirement, but allows tradeoffs between subgroup mix and the journeyman component of the career and first term forces. Additionally, the modification permits evaluation of tradeoffs between subgroup mix and the distribution of the career force by year of TAFMS.

The basic model is exercised for each CFS. It assumes that attrition is dependent on years of TAFMS, as well as membership in a given CFS. Additionally, it assumes the time (in terms of TAFMS) that it takes to upgrade to the supervisor/technician skill level is dependent on CFS membership. The functional relationship involved is not specified. Rather, the assumption is implicit by introducing attrition rates by year of TAFMS and CFS and upgrade rates by CFS. The proposed modification additionally implies that attrition and upgrade times to the 7 skill level are dependent on membership in some subgroup within the CFS. Within the subgroup, it is assumed that attrition for 7 and 9 personnel is the same as for 5 level careerists with the same years of TAFMS.

Finally, the most recent empirical data is used to postulate a stable career force representation for the future. While historical attrition and upgrade rates are probably achievable, there is less certainty that they are, in fact, objective rates. This point cannot be overstated. Certainly, the

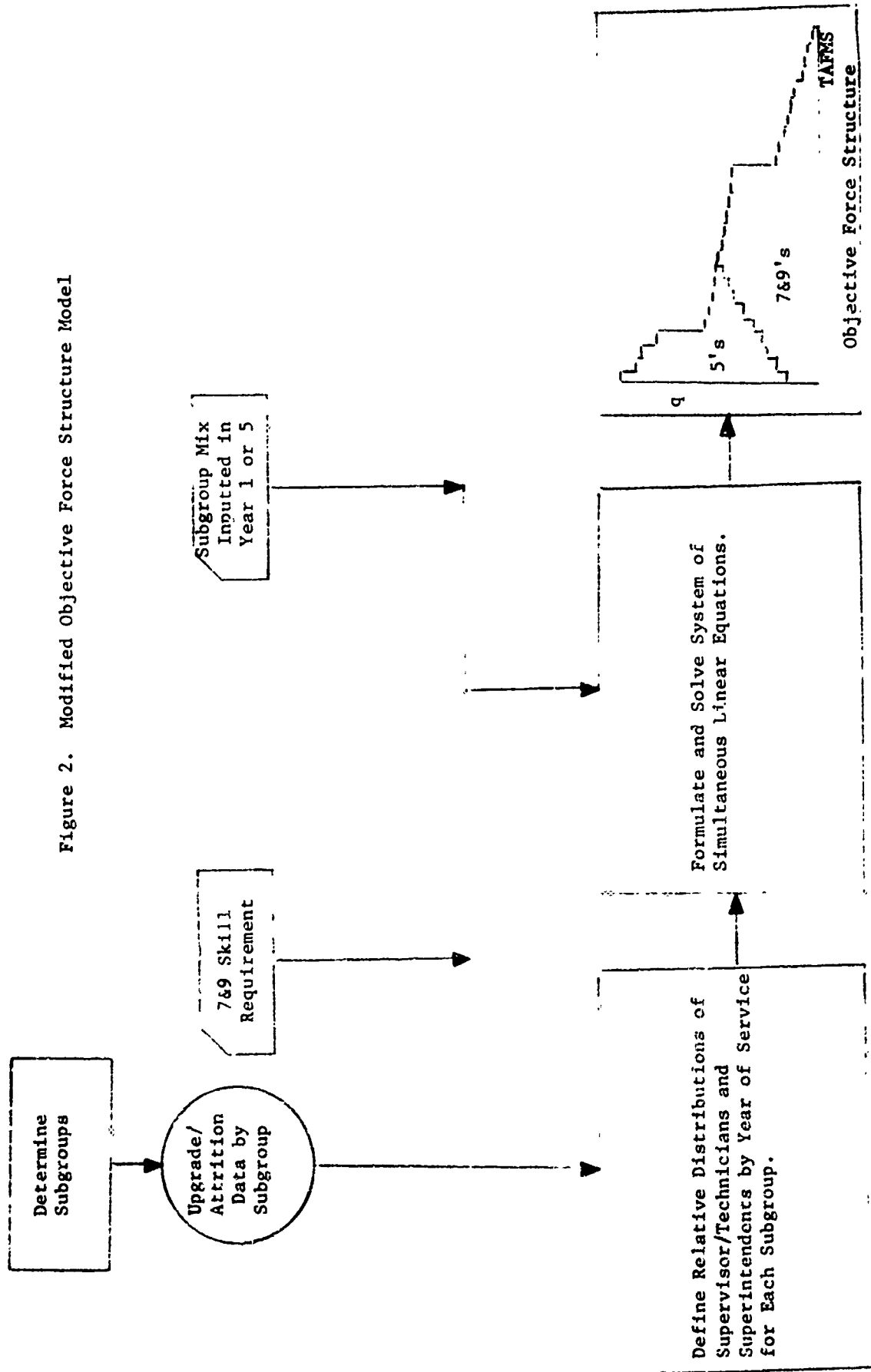


Figure 2. Modified Objective Force Structure Model

Figure 2. Modified Objective Force Structure Model.

objective nature of these rates, the supervisor/superintendent requirement, and the specified force range each critically influence the desirability of the resultant force configuration.

While these assumptions are quite restrictive, two things should be kept in mind. First, the model is not meant to provide precise measurements; it is an aggregate planning model used to represent alternative stationary states and to illustrate dramatic differences in alternative states. Secondly, the model is normally used in conjunction with other planning models which take into account many of the restrictions mentioned.

Use/Limitation

The principal use of the modified Objective Force Structure Model would be an evaluation of alternative stationary force objectives as a function of a specified subgroup mix. The question which prompted this research effort was the effect of "accession quality" on the TOPCAP objective configuration. Figure 3 illustrates a flow diagram representing one general approach to the problem. Objective force structures, based on aptitude mixes resulting from alternative applicant pools could be postulated and evaluated.

Additional analyses might include: (1) hypothesizing upgrade and attrition data for personnel with aptitude scores below the present cutoff, and evaluating the effects on the force structure objective of reducing the cutoff score; (2) evaluating the effects of alternative mixes on the force

⁶Although the computer-based methodologies in TOPCAP are in constant flux, this general aspect of the model may prove useful in the future. A growing concern that the free-flow nature of upgrade rates is being biased by restrictions imposed by the promotion system and a greater confidence in establishing "objective" promotion rates, as opposed to "objective" upgrade rates, suggest an alternative use for the model. In such a case, retention would remain as before—a principal variable. The achievement rates for the subgroups would then be based on objective promotion patterns, not upgrade patterns. The requirement would be in the form of E8s and E9s, not the number of 7 and 9 skill level personnel. With the implementation of the up or out policy, at some point, everyone would have been promoted to at least E8 or left the service.

⁷The Training Line Simulator is an entity simulation model which simulates the flow of airman recruits through basic military training (BMT) and initial post BMT training. See AFHRL-TR-72-69, *Airman Training Line Simulator*, and AFHRL-TP-73-61, *United States Air Force Training Line Simulator*.

structure for Air Force Specialty Codes (AFSCs) with more than one selector area; and (3) determining any configuration when the achievement rates for the subgroups are known, a given requirement is specified for those who possess the characteristic in question, knowledge that at some point everyone has achieved that characteristic, and the subgroup mix is specified at some point in time.⁶

The principal limitation of the proposed modification involves the number of subgroups identified, and is reflected in two manners. As the number of subgroups increases, the distortion due to "rounding" increases. Due to the aggregate nature of the model, when the number of subgroups is not excessive, "rounding" to achieve whole numbers of personnel is not a significant problem. Secondly, as the subgroup categories increase, the number of equations in the system of simultaneous linear equations increases, affecting the computing time involved in the solution process.

Relationship to Other Models

As previously mentioned, Volume VI and the Annexes to Volume III of the Air Force Personnel Plan describe in detail the relationship between the Objective Force Structure Model and other models in the TOPCAP Computerized Management System and their role in the development of the TOPCAP objective grade and force configuration, as well as enlisted force programming actions. The following discussion will focus on the effects of the proposed modifications on that relationship.

Inputs to the Objective Force Structure Model come from two sources. First, the 7 and 9 skill level requirement is generated from the manpower authorization file and the TOPCAP Skill Projection Model. Upgrade and attrition data is inputted by CFS by year of Total Active Federal Military Service. The modified model would accept input from the manpower authorization file and Skill Projection Model in the same form as the basic model. Upgrade and attrition data inputted to the modified model would be broken out by an additional dimension (defined by the subgroups). It would also be necessary to specify the subgroup mix inputted to the CFS, in either year 1 or year 5. This data might be inputted from a model such as the Air Force Training Line Simulator, or specified through some other means.⁷

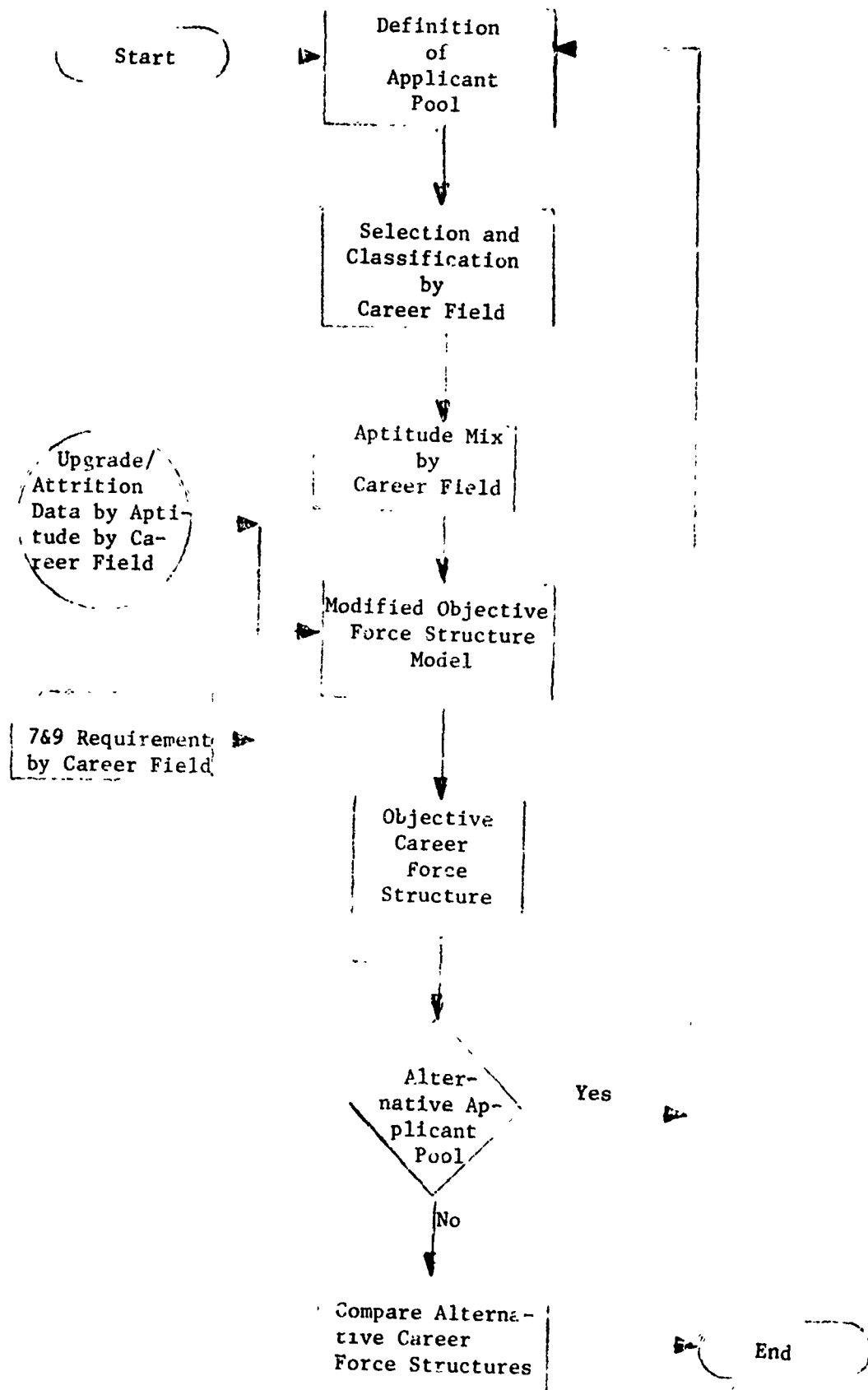


Figure 3. Method for assessing effects of accession quality on objective force structure.

Although the modification allows analysis by subgroup within the CFS, the output is aggregated and displayed as a career force configuration: distribution by year of TAFMS of total personnel in the career force, as well as a distribution of 7 and 9 skill level personnel. This output of the modified model takes the same form as the basic Objective Force Structure Model.

Compatibility among models is maintained through a commonality in attrition rates. Since the modification, in effect, weights empirical attrition rates according to the subgroup mix at each year of service, this circumstance would have to be resolved in loss rate analyses.⁸

IV. CONCLUSION

This technical report suggests a modification to one of the methodologies in the TOPCAP Computerized Management System, the Objective Force Structure Model. The change allows for disaggregation beyond the occupational context in development of enlisted force structure objectives.

⁸Section Four, "Airman Programming," Volume Six, *Personnel Programming Section*, p. 4-4.

The effort was prompted by a need for a method to evaluate the trade-offs between the 'quality' of personnel entering the career force and the effects on the TOPCAP objective grade and force configuration. While the original intent was to evaluate alternative aptitude mixes, the modification is general in the sense that it allows user-specified disaggregation within the career field subdivision. The disaggregation might take the form of any mutually exclusive and totally exhaustive breakout within the CFS (groups defined by aptitude score, by education, completion of some specific course of study, etc.).

An effort was made to maintain the basic premises involved in the development of TOPCAP, to insure ease of implementation and compatibility with other models in the TOPCAP Computerized Management System. As a result, the model remains an aggregate planning model used to identify structural differences in postulated alternative stationary states and is meant to be used as one of many inputs in the formulation of an enlisted force management system.

A subsequent technical report will utilize the modified Objective Force Structure Model in an analysis of the effects of alternative enlisted aptitude mixes on the force structure objective.

REFERENCES

- Fisher, F.M. Aspects of cost-benefit analysis in defense manpower planning. In N.A.B. Wilson (Ed.), *Manpower research in a defense context*. New York: American Elsevier Publishing Company, Inc., 1969.
- Hatch, R.S., Pierce, M., Nauta, F., & Zimmer, C.E. *Airman training line simulator*. AFHRL-TR-72-69, AD-754 851. Lackland AFB, Tex.: Personnel Research Division, Air Force Human Resources Laboratory. August 1972.
- Nauta, F., & Pierce, M. *United States Air Force training line simulator*. AFHRL-TR-73-61, AD-774 572. Lackland AFB, Tex.: Manpower and Personnel Systems Division, Air Force Human Resources Laboratory, December 1973.
- USAF Personnel Plan. *Volume one, personnel management objectives*. Washington, D.C.: Department of the Air Force, 1 June 1971.
- USAF Personnel Plan. *Volume three, airman structure (TOPCAP)*. Washington, D.C.: Department of the Air Force, 1 June 1971.
- USAF Personnel Plan. *Volume three, airman structure annexes (Draft)*. Washington, D.C.: Department of the Air Force, 1 January 1973.
- USAF Personnel Plan. *Volume six, personnel programming system, Part II, Section Two, Airman Programming*. Washington, D.C.: Department of the Air Force, 1 April 1971.

APPENDIX A: MATHEMATICAL MODELS

Objective Force Structure Model

Indexing Notation

- i : Index denoting year of total active federal military service
- m : Denotes the year in which 100% level upgrading is achieved
- n : Denotes the first year of service in which 7 level upgrading occurs

Known Parameters (based on empirical data)

- a_i : Represents the attrition rate in year i , $i = 1, 30$. That is, the proportion of individuals who fail to continue to the next year group of service. Thus $0 \leq a_i \leq 1$.
- b_i : Represents the cumulative upgrade rate to the 7 skill level for year i , $i = n, m$. The first year upgrade to the 7 level occurs is n . By year m , 100% upgrade to the 7 level has been achieved. Since b_i represents a cumulative rate, $b_m = 1$ and b_i 's ($i = n, m-1$) have some value such that $0 < b_i \leq 1$ and $b_i \leq b_{i+1}$.
- c_i : Represents a survival rate. Since by year m everyone has achieved at least the 7 skill level or exited the Air Force, c_i is used to represent 7 and 9 skill level personnel (total personnel) in subsequent years as a proportion of year m . Thus, $c_i = \prod_{k=m}^{i-1} (1-a_k)$ for $i = m+1, 30$.

Specified Parameters

- d : Represents the number of 7 and 9 level requirements.

Unknown Parameters

- X_i : Represents the 7 and 9 population in year i , $i = n, 30$.
- Z_i : Represents the total population in year i , $i = 1, 30$.

Procedure

The relative distribution of 7 and 9 level personnel by total active federal military service is defined by the b_i 's and the c_i 's. The total number of 7 and 9 level personnel required is specified, d , and fit to the defined distribution. The proportion of 7 and 9 personnel in year m can be written as

$$\frac{1}{(\sum_{i=n}^m b_i + \sum_{i=m+1}^{30} c_i)}, \text{ thus } X_m = d \left[\frac{1}{(\sum_{i=n}^m b_i + \sum_{i=m+1}^{30} c_i)} \right].$$

The remaining years n through $m-1$ and $m+1$ through 30 can be calculated as a proportion of X_m . $X_i = b_i X_m$ for years $i = n, m-1$ and $X_i = c_i X_m$ for years $i = m+1, 30$. Additionally, the total force by total active federal military service can be calculated. Since only 7 and 9 level personnel remain in year 30, $Z_{30} = X_{30}$. Total force by year of service is calculated as $Z_i = [Z_{i+1}/(1-a_i)]$, $i = 1, 29$. As previously discussed, $Z_i = X_i$ for $i = m, 30$.

Modified Objective Force Structure Model

Premise

There exists J mutually exclusive and totally exhaustive identifiable subgroups.

Indexing Notation

- i : Index denoting year of total active federal military service
- j : Index denoting subgroup membership

$m(j)$: Denotes year 100% 7 level upgrading is achieved for subgroup j

$n(j)$: Denotes the first year of service in which 7 level upgrading occurs in subgroup j

Known Parameters (based on empirical data)

a_{ij} : Represents the attrition rate in year i , $i = 1, 30$ for subgroup j , $j = 1, J$. That is, the proportion of subgroup j in year i who fail to continue to the next year of service. Thus $0 \leq a_{ij} \leq 1$.

b_{ij} : Represents the cumulative upgrade rate to the 7 skill level in year i , $i = n(j), m(j)$ for subgroup j , $j = 1, J$. The first year upgrade to the 7 level occurs for subgroup j is $n(j)$. By year $m(j)$, 100% upgrade to the 7 level has been attained in subgroup j . Since b_{ij} represents a cumulative rate, $b_{m(j),j} = 1$, for each subgroup $j = 1, J$, and b_{ij} 's ($i = n(j), m(j)-1$) have values such that $0 \leq b_{ij} \leq 1$ and $b_{ij} \leq b_{i+1,j}$.

c_{ij} : Represents a survival rate. Since by year $m(j)$ everyone in subgroup j has achieved at least the 7 level or exited the Air Force, c_{ij} represents 7 and 9 level personnel (subgroup j) in subsequent years, as a proportion of year $m(j)$. For all $j = 1, J$

$$c_{ij} = \prod_{k=m(j)}^{i-1} (1 - a_{kj}) \text{ for } i = m(j)+1, 30$$

Specified Parameters

d : Represents the number of 7 and 9 level requirements

p_j : Represents the proportion of the first year force in subgroup j . Since the J groups are mutually exclusive and totally exhaustive,

$$\sum_{j=1}^J p_j = 1.$$

Unknown Parameters

X_{ij} : Represents the 7 and 9 population in year i , $i = n(j), 30$ for subgroup j , $j = 1, J$

Y_j : Represents the portion of 7 and 9 level requirements in subgroup j that would result given historical upgrade and loss rates and the quality group distribution in the first year of service

Z_{ij} : Represents the total population in year i , $i = 1, 30$, subgroup j , $j = 1, J$

Procedure

The b_{ij} 's and c_{ij} 's define a relative distribution of 7 and 9 level personnel by active federal military service for each subgroup j . If the portion of 7 and 9 level requirements in subgroup j were known, Y_j , 7 and 9 level populations, as well as total populations, could be calculated by year by subgroup needed to sustain these requirements. As

$$X_{m(j),j} = Y_j \left[1 / \sum_{i=n(j)}^{m(j)} b_{ij} + \sum_{i=m(j)+1}^{30} c_{ij} \right]$$

for years $n(j)$ to $m(j)-1$

$$X_{ij} = b_{ij} X_{m(j),j}$$

and for years $m(j)+1$ to 30

$$X_{ij} = c_{ij} X_{m(j),j}$$

The Y_j 's are not known and must be determined. The total population in year 30, for each subgroup j , can be written

$$X_{30,j} = Z_{30,j} = Y_j \left[b_{30,j} / \sum_{i=n(j)}^{m(j)} b_{ij} + \sum_{i=m(j)+1}^{30} c_{ij} \right]$$

for all $j = 1, J$

By successively applying $Z_{ij} = Z_{i+1,j}/(1-a_{ij})$, the total population, for each subgroup in year (1), can be expressed

$$Z_{1j} = Y_j [b_{30,j}/(\sum_{i=n(j)}^{m(j)} b_{ij} + \sum_{i=m(j)+1}^{30} c_{ij})]/\pi (1-a_{ij})$$

Thus, the proportion of the first year population in subgroup j can be expressed.

Let

$$f_j = [b_{30,j}/(\sum_{i=n(j)}^{m(j)} b_{ij} + \sum_{i=m(j)+1}^{30} c_{ij})]/\pi (1-a_{ij})$$

thus

$$p_j = f_j Y_j / \sum_{j=1}^J f_j Y_j$$

Since $p_J = 1 - \sum_{j=1}^{J-1} p_j$ this provides J-1 unique linear equations. Additionally, $\sum_{j=1}^J Y_j = d$; that is, the total 7 and 9 level requirement must be distributed across the totally exhaustive and mutually exclusive subgroups. Thus:

$$p_j \sum_{k=j+1}^J f_k Y_k + f_j (p_j - 1) Y_j = 0 \quad \text{for } j=1$$

$$p_j \sum_{k=1}^{j-1} f_k Y_k + p_j \sum_{k=j+1}^J f_k Y_k + f_j (p_j - 1) Y_j = 0 \quad \text{for } j=2, J-1$$

and

$$\sum_{j=1}^J Y_j = d$$

After solving the system for simultaneous equations and determining the Y_j 's, the 7 and 9 level personnel distribution by year of service can be calculated.⁹

$$X_{m(j),j} = Y_j [1/(\sum_{i=n(j)}^{m(j)} b_{ij} + \sum_{i=m(j)+1}^{30} c_{ij})]$$

$$X_{ij} = b_{ij} X_{m(j),j} \quad \text{for } i = n(j), m(j)$$

$$X_{ij} = c_{ij} X_{m(j),j} \quad \text{for } i = m(j)+1, 30$$

Aggregating across subgroup determines the 7 and 9 level personnel distribution based on a defined 7 and 9 level requirement, initial (1st year) subgroup mix, and empirical upgrade and attrition data.

$$X_i = \sum_{j=1}^J X_{ij} \quad , \quad i = 1, 30$$

⁹This is the simplest case of a set of nonhomogeneous linear equations. There are J equations and J unknowns. When the equations are all linearly independent and consistent, a unique solution exists.

Since $Z_{30j} = X_{30j}$

total populations can be calculated for each subgroup by year of service by successively applying

$$Z_{ij} = Z_{i+1,j} / (1 - a_{ij}) \quad \text{for } i = 1, 29 \text{ and } j = 1, J$$

Aggregating across subgroup determines the total force by year of service needed to sustain a given 7 and 9 level requirement and based on a given subgroup in the first year of service.

$$Z_i = \sum_{j=1}^J Z_{ij} \quad i = 1, 30.$$

Comment

The subgroup mix p_j is specified in year 1. It could be specified in any year without affecting the logic of the modification. Practically, year 1 or year 5 would be used.